

Progression through written Calculation

Addition and Subtraction

ADDITION

STAGE 1

Children will use practical equipment to combine two groups of objects and find the total.

Children will begin to understand that addition can be done in any order – it is commutative.

Children will represent calculations using objects and talk about their representations.



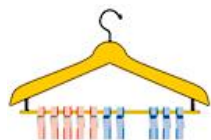
Fingers



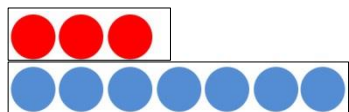
Cubes



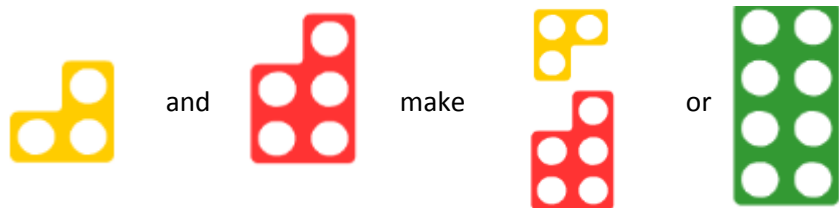
Bead String



Pegs



Counters



SUBTRACTION

STAGE 1

Children will use practical equipment to physically remove an amount from the group and find the total remaining.

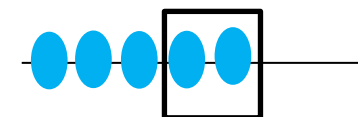
Children will represent calculations using objects and talk about their representations.



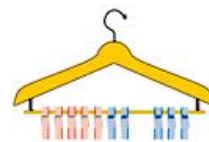
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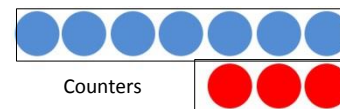
Cubes



Bead String



Pegs



Counters



Children will also be introduced to the language of comparison including equal use of the vocabulary 'less' and 'more'.

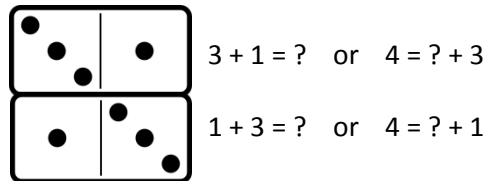
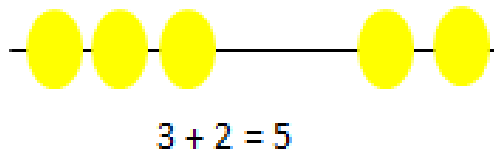
There are more blue counters than red counters. 

There are less red counters than blue counters. 

STAGE 2

Practical equipment will continue to support children's development of mental pictures and images.
 Children will begin to represent their mental images and practical equipment using pictures.
 Children will begin to use number sentences alongside their pictures and practical equipment.
 The direct link between addition and subtraction will be made explicit using models and representations.
 Children will use number lines to represent their mental images.

Pictorial Methods

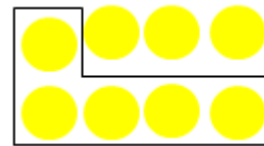


How many flowers are there altogether in these vases?

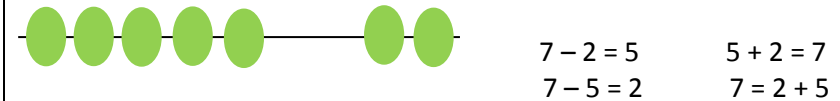
STAGE 2

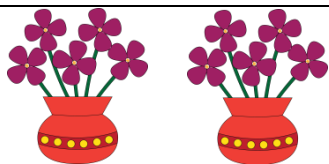
Practical equipment will continue to support children's development of mental pictures and images.
 Children will begin to represent their mental images and practical equipment using pictures.
 Children will begin to use number sentences alongside their pictures and practical equipment.
 Children will use number lines to represent their mental images.
 Children will understand that subtraction is not commutative and so the numbers can be in any order but will result in a different answer.
 The direct link between addition and subtraction will be made explicit using models and representations.

Pictorial Methods



$8 + ? = 10$ $? + 8 = 10$
 $10 - 8 = ?$ $10 - 8 = 2$





$$5 + 5 = 10$$

There are 5 apples on a tree. If 2 fall how many will be left?

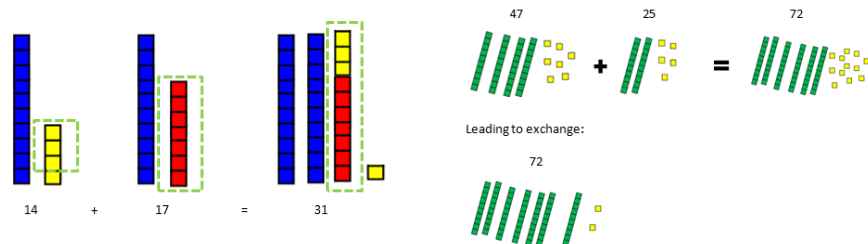


$$5 - 2 = 3$$

STAGE 3

Children will feel confident when using practical equipment to help them combine groups of objects with numbers up to 20. They will continue to use practical equipment and use number lines and hundred squares to support their mental methods. Children will start to work with totals greater than 20 which require them to apply their knowledge of the principle of exchange.

Concrete Method

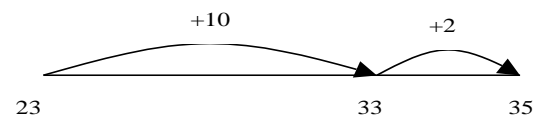


Pictorial Methods

Continue to use number lines to develop understanding of:

Counting on in tens and ones

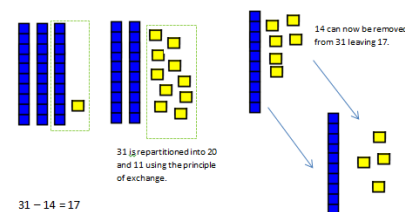
$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$



STAGE 3

Children will now be confident when using concrete equipment to help them 'take away' and 'find the difference'. They will continue to use practical equipment and use number lines and hundred squares to support their mental methods. Children will start to work with totals greater than 20 which require them to apply their knowledge of the principle of exchange.

Concrete Methods



Pictorial Methods

Continue to use number lines to model take-away and difference.



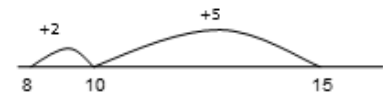
Some children may also benefit from being taught different methods of using number lines such as:

Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of ten.

e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

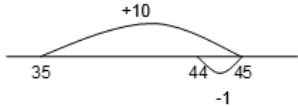
$8 + 7 = 15$



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

$35 + 9 = 44$



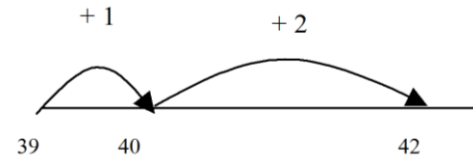
Using a 100 square:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$14 + 13 = ?$

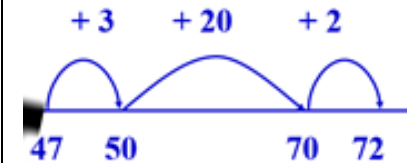
$14 + 10 = 24$

$24 + 3 = 27$



The difference between 39 and 42 is 3.

The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



Using a 100 square:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

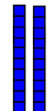
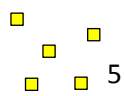
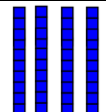

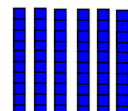
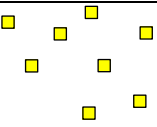
$25 - 12 = ?$

$25 - 10 = 15$

$15 - 2 = 13$

STAGE 4

Children will now begin to organise their concrete equipment in a vertical manner where their combined totals are situated at the bottom (within 10). Children use the expanded written method.

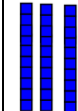
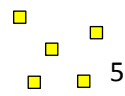
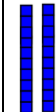
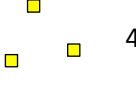


Tens	Ones
 20	 5
 40	 3
	

$$\begin{array}{r} 20 + 5 \\ 40 + 3 \\ \hline 60 + 8 = 68 \end{array}$$

This method should still be used alongside blank number lines and 100 squares.

STAGE 4

They will begin to organise their concrete equipment in a vertical manner where the amount that remains at the end of the calculation is situated at the bottom (within 10). Children use the expanded written method.

Tens	Ones
 30	 5
 20	 4
	

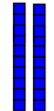
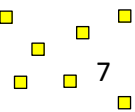
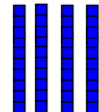

$$\begin{array}{r} 30 \ 5 \\ - 20 \ 4 \\ \hline 10 + 1 = 11 \end{array}$$

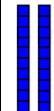
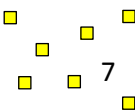
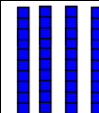
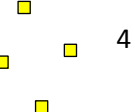
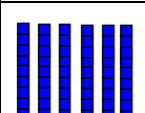
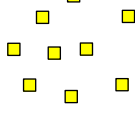
This method should still be used alongside blank number lines and 100 squares.

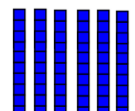
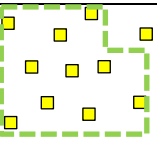
STAGE 5

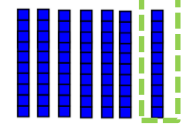

They will now begin to organise their concrete equipment in a vertical manner where their combined totals are situated at the bottom

$27 + 44 =$

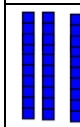

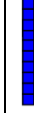

Tens	Ones
 20	 7
 40	 4

Tens	Ones
 20	 7
 40	 4
 60	 11

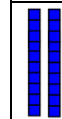


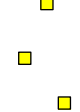

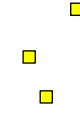
Tens	Ones
20	7
40	4
 60	 11

Tens	Ones
20	7
40	4
 70	1 

$31 - 14 =$

Tens	Ones
	
	



Tens	Ones
 3	 11
	
	

STAGE 6

Children will have a full understanding of the links between the concrete representation for column addition and the formal written method. They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r}
 27 \\
 + 44 \\
 \hline
 71 \\
 \hline
 1
 \end{array}$$

STAGE 7

Children will have a full understanding of the links between the concrete representation for column addition and the formal written method. They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r}
 \overset{2}{\cancel{3}}\overset{1}{1} \\
 - 14 \\
 \hline
 17 \\
 \hline
 \end{array}$$

Multiplication and Division

MULTIPLICATION

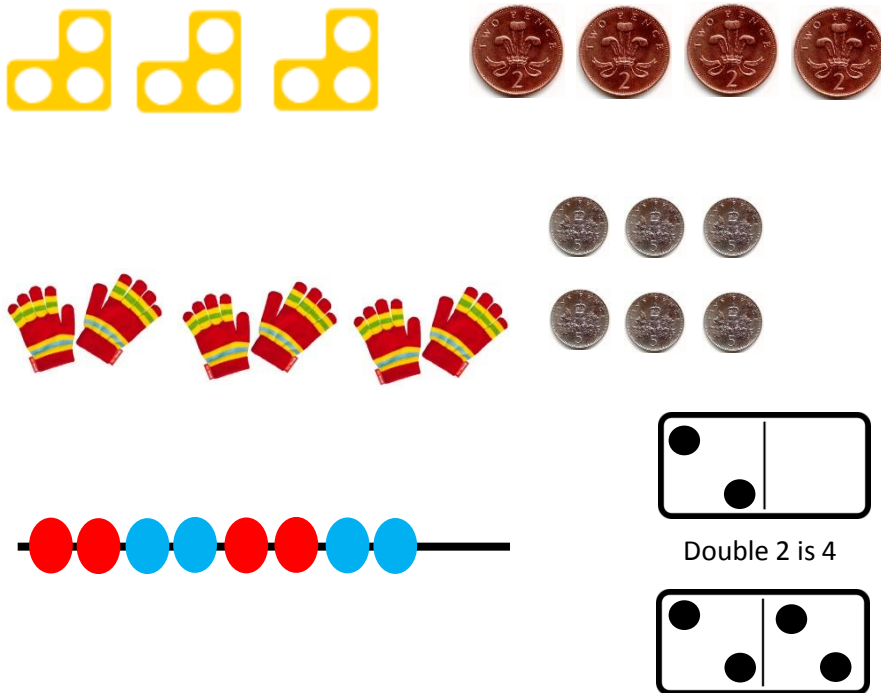
STAGE 1

Children will experience practical opportunities involving equal sets or groups using a variety of equipment. Practical support will support children's development of mental pictures and images.

Children will begin to orally count in different multiples including twos, fives and tens making links to everyday groupings (below) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources.

They will begin to use the language and associated representations of doubling.



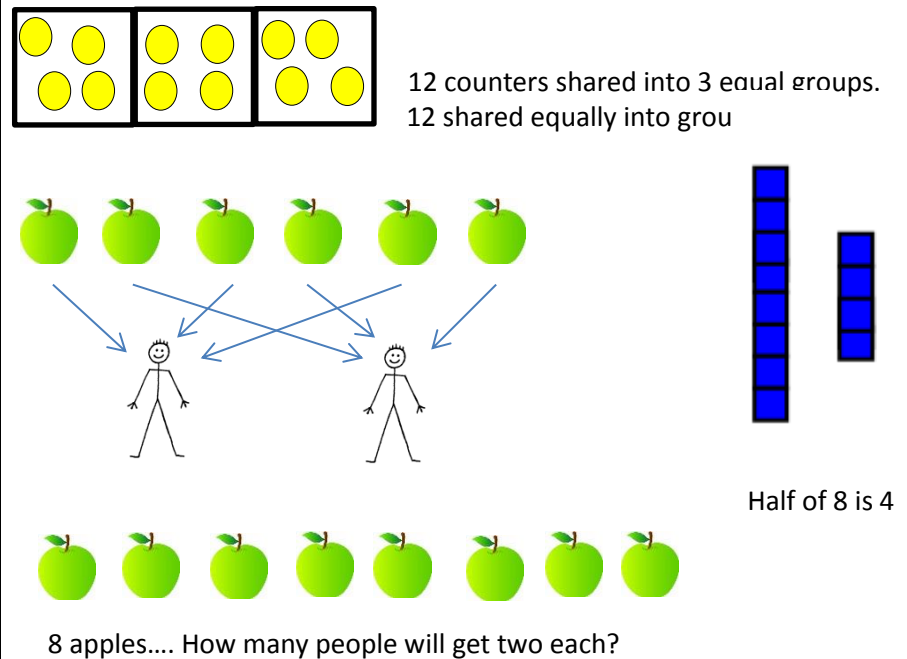
DIVISION

STAGE 1

Children will explore the language of sharing. Children will experience practical activities in sharing objects between a small number of groups or people with the emphasis on sharing equally.

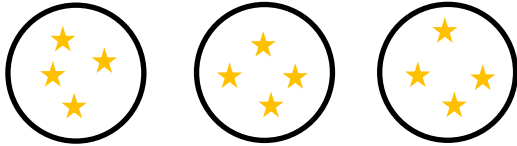
Alongside this and with the same weighting children should be introduced to grouping objects as a representation of division with the emphasis on equal groups.

They will begin to use language and associated representations of halving. Children are encouraged to develop ways of recording their findings using pictures.



STAGE 2

Children will begin to arrange objects into equal groups to aid counting.

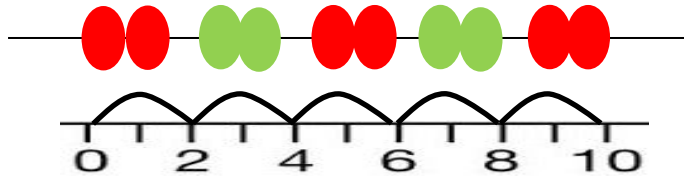
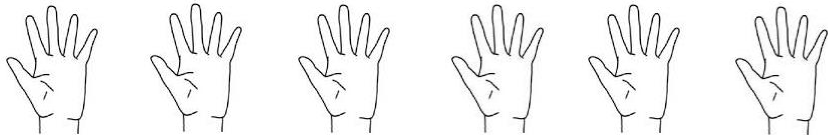


They will continue to count in multiples and begin to relate this to multiplication through finger counting.



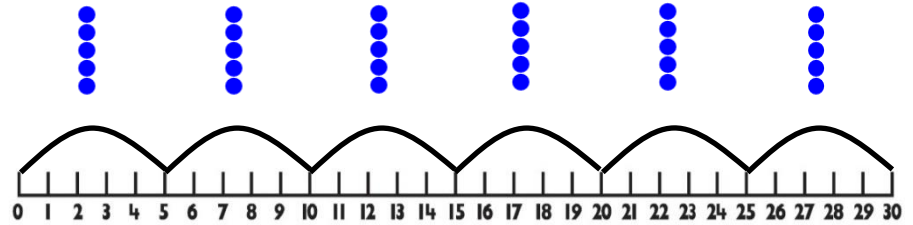
"3" "6" "9" "12" "15"

Children will be introduced to a variety of representations of repeated addition; they will see these representations alongside each other and begin to make connections between them.



STAGE 2

Children will relate the grouping of objects related to repeated subtraction and begin to represent this using a number line whilst continuing to use concrete equipment.



How many 2s are in 10?



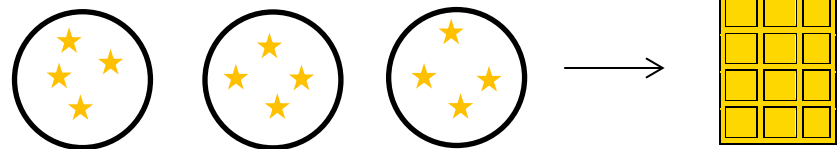
How many 2s can we take away from 10?

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this is the inverse of multiplication.



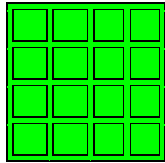
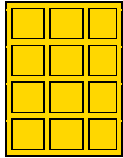
"3" "6" "9" "12" "15"

Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into arrays.

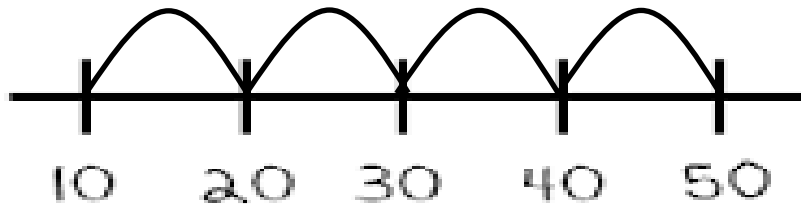


STAGE 2 CONTINUED

Children will be introduced to the array using concrete equipment for small numbers as a way of organising groups to show repeated addition and commutativity. Everyday objects such as egg boxes can be used to support this.



Counting in tens from zero.



STAGE 2 CONTINUED

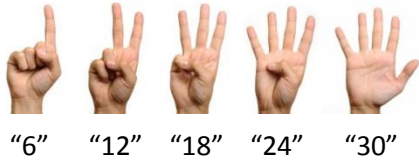
The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line so $p \div q$ can be written as $\frac{p}{q}$

1 ÷ 2 can be written as $\frac{1}{2}$

STAGE 3

Children will continue to count in multiples and relate this to multiplication through finger counting.



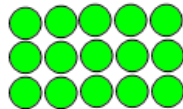
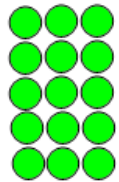
They will be able to model a calculation using a practical array which demonstrates an effective method of counting and link to repeated addition. Children will need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.



$3 \times 4 = 12$
 $4 \times 3 = 12$

$6 \times 2 = 12$
 $2 \times 6 = 12$

$1 \times 12 = 12$
 $12 \times 1 = 12$

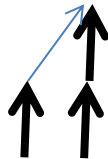


$3 + 3 + 3 + 3 + 3 = 15$

$5 + 5 + 5 = 15$

Children should feel confident with language related to scaling when talking about multiplication.

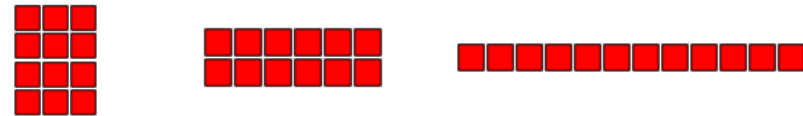
"Twice as big."



STAGE 3

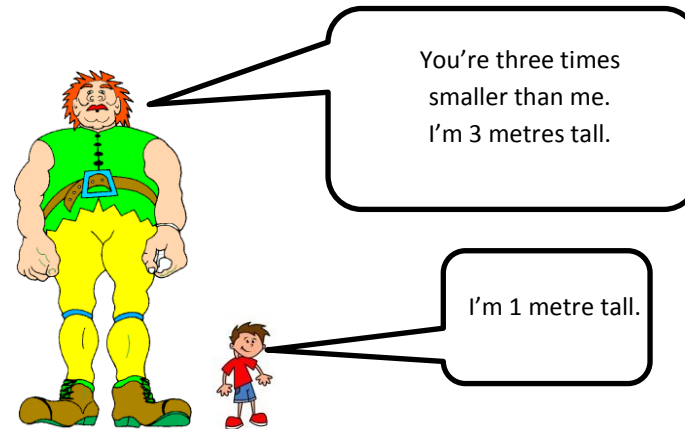
Children will use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

Children will build on their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts of a given number by making a variety of arrays and explaining what they show.



12 into ___ equal groups gives ___ in each group.
12 into equal groups of ___ gives ___ groups.

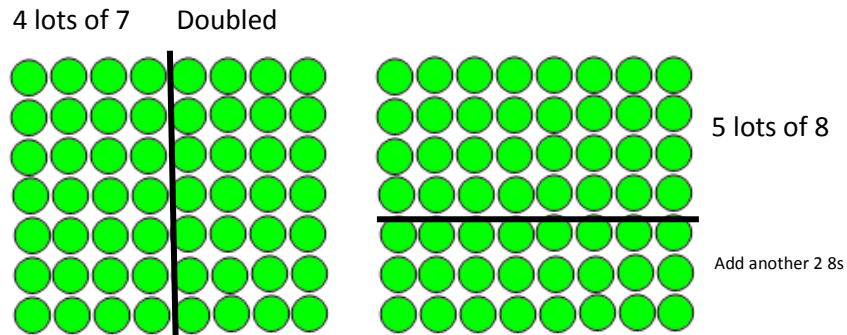
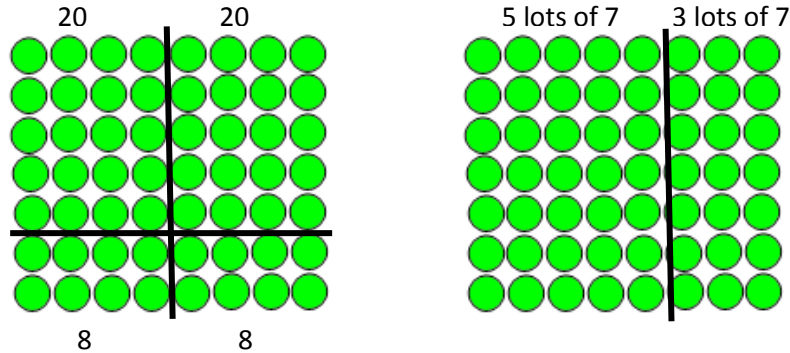
The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (eg. Half the size).



STAGE 4 - MULTIPLICATION

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. Eg for 7 X 8.... Children may find counting in 7s and 8s tricky but they can look for friendly numbers which are easier to calculate eg. 4X5, 4X2, 4X5, 4X2

Think flexibly about 7X8

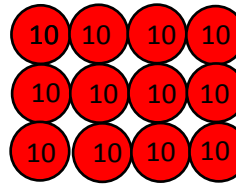


Children should continue to experience the language of scaling (eg. Scaling up pictures by multiplying by powers of 10, multiplying by powers of 1000 in converting between units of measure.

STAGE 4 - DIVISION

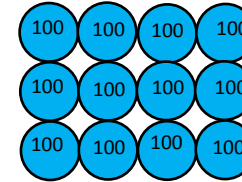
Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?'

120 ÷ 3



120 shared equally between 3 is 40
 120 shared equally between 4 is 30
 3 equal groups of 40 make 120
 4 equal groups of 30 make 120

1200 ÷ 3



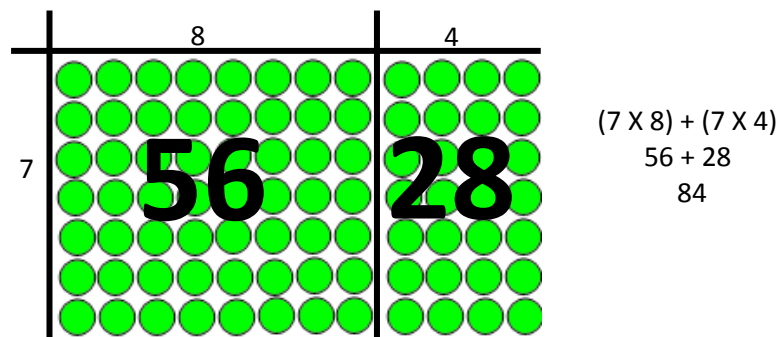
1200 shared equally between 3 is 400.
 1200 shared equally between 4 is 300
 3 equal groups of 400 make 1200
 4 equal groups of 300 make 1200

STAGE 5 – MULTIPLICATION

Children will continue to work with arrays, exploring larger numbers, leading into the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with grid lines. When appropriate, children should move to using the grid showing the numbers only.

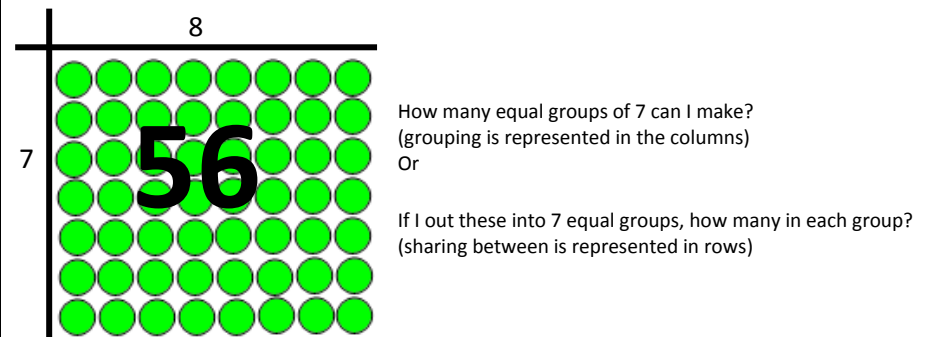
Children should begin using grid method for two and three digit by one digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.



X	8	4
7	56	28

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for both sharing between or equal groups of where the dots within the array each represent 1.



Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

